

Simplify. Use absolute value signs when necessary.

$$\begin{array}{c} 24 \\ | \quad | \\ 12 \cdot 2 \\ | \quad | \\ 3 \cdot 4 \cdot 2 \\ | \quad | \\ 3 \cdot 3 \cdot 2 \end{array}$$

$$\begin{array}{c} 162 \\ | \quad | \\ 81 \cdot 2 \\ | \quad | \\ 9 \cdot 9 \cdot 2 \\ | \quad | \quad | \\ 3 \cdot 3 \cdot 3 \cdot 2 \end{array}$$

$$\begin{array}{c} 128 \\ | \quad | \\ 64 \cdot 2 \\ | \quad | \\ 8 \cdot 8 \cdot 2 \\ | \quad | \quad | \\ 2 \cdot 4 \cdot 4 \cdot 2 \\ | \quad | \quad | \\ 2 \cdot 2 \cdot 2 \cdot 2 \end{array}$$

1) $\sqrt{24} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3}$
 $= 2\sqrt{2 \cdot 3} = 2\sqrt{6}$

* neg #'s can be under an odd root
 (bring out as -1)
 3) $\sqrt[3]{-162} = 3(-1)\sqrt[3]{2 \cdot 3}$
 $= -3\sqrt[3]{6}$

5) $\sqrt[4]{128n^8} \rightarrow 2 \text{ groups of 4}$
 $\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} \\ 2 \cdot n \cdot n \sqrt[4]{2 \cdot 2 \cdot 2} = 2n^2\sqrt[4]{8}$

7) $\sqrt[3]{224r^7} \rightarrow 1 \text{ group of 5}, 2 \text{ left over}$
 $\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7} \\ 2 \cdot r \sqrt[3]{7 \cdot r \cdot r} = 2r\sqrt[3]{7r^2}$

9) $\sqrt{392x^2} \rightarrow 1 \text{ group of 2}$
 $\sqrt{2 \cdot 2 \cdot 2 \cdot 7 \cdot 7} \\ 2 \cdot 7 \cdot x \sqrt{2} = 14x\sqrt{2}$

11) $\sqrt[4]{405x^3y^2} \rightarrow \text{No groups of 4}$
 $\sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5} \\ 3\sqrt[4]{5x^3y^2}$

13) $\sqrt[4]{128x^7y^7} \rightarrow 1 \text{ group of 4}, 3 \text{ left over}$
 $\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} \\ 2 \cdot x \cdot y \sqrt[4]{2 \cdot 2 \cdot 2 \cdot x^3y^3} \\ 2xy\sqrt[4]{8x^3y^3}$

15) $\sqrt[3]{448x^7y^7}$
 $\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7} \\ 2xy\sqrt[3]{7xy}$

Critical thinking questions:

17) What simplifies into $2mn^2\sqrt[3]{5mn^2}$?
 * raise to the 3rd power to move back under the radical

$$\begin{aligned} & \sqrt[3]{2^3 \cdot 5 \cdot m^3(n^2)^3 \cdot n^2} \\ & \sqrt[3]{8 \cdot 5 \cdot m^4 \cdot n^6 \cdot n^2} \\ & \boxed{3\sqrt[3]{40m^4n^8}} \end{aligned}$$

2) $\sqrt[3]{1000} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5}$
 $2 \cdot 5 = \boxed{10}$ Perfect cube

4) $\sqrt{512} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$
 $= 2^4\sqrt{2} = \boxed{16\sqrt{2}}$

6) $\sqrt{98k} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot k}$
 $= 2^4\sqrt{3 \cdot k} = \boxed{4\sqrt{16k}}$

8) $\sqrt[3]{24m^3} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3}$
 $= \boxed{2m\sqrt[3]{3}}$

10) $\sqrt{512x^7} = \sqrt{2 \cdot 2 \cdot 2}$
 $= 2^4\sqrt{2} = \boxed{16x\sqrt{2}}$

12) $\sqrt[3]{-16a^3b^8} \rightarrow 1 \text{ group of } 3$
 $-1 \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$
 $-1 \cdot 2 \cdot a \cdot b^2 \sqrt[3]{2b^6}$
 $= \boxed{-2ab^2\sqrt[3]{2b^6}}$

14) $\sqrt[3]{16xy} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot x \cdot y}$
 $= \boxed{2\sqrt[3]{2xy}}$

16) $\sqrt[3]{56x^5y} \rightarrow 1 \text{ group of } 3$
 $\text{with } 2 \text{ left over}$
 $\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 7}$
 $= \boxed{2x\sqrt[3]{7x^2y}}$

18) Simplify $\sqrt[4]{3 \cdot 2^n \cdot x^{2n}y^{n+3}} \rightarrow y^{\frac{n}{4}} \cdot y^{\frac{3}{4}}$ (1 group of n)
 $\downarrow x^n \cdot x^n \text{ (2 groups of n)}$
 $\text{exp. matches (1 group of n)}$
 $= \boxed{2x^2y\sqrt[4]{3y^3}}$