Writing Transformations of Quadratic Functions

The lowest point on a parabola that opens up or the highest point on a parabola that opens down is the **vertex**. The **vertex form** of a quadratic function is $f(x) = a(x - h)^2 + k$, where $a \neq 0$ and the vertex is (h, k).

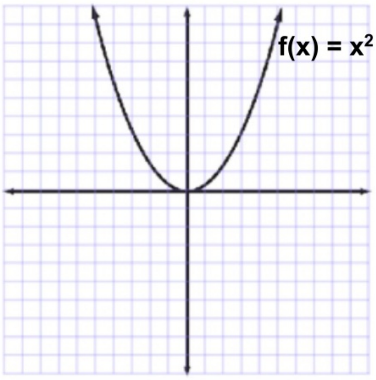
$$f(x) = a(x - h)^2 + k$$

a indicates a reflection in the *x*-axis and/or a vertical stretch or shrink.

h indicates a horizontal translation.

k indicates a vertical translation.

Parent Function



Example:

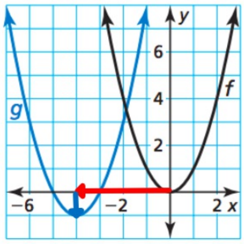
Describe the transformation of $f(x) = x^2$ represented by $g(x) = (x + 4)^2 - 1$. Then graph each function.

SOLUTION

Notice that the function is of the form $g(x) = (x - h)^2 + k$. Rewrite the function to identify h and k.

$$g(x) = (x - \underset{h}{(-4)})^2 + \underset{k}{(-1)}$$

▶ Because $h = -4$ and $k = -1$, the graph of g is a translation 4 units left and 1 unit down of the graph of f.



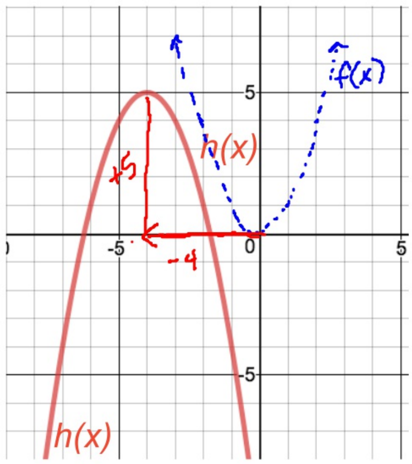
Examples:

1. Describe the transformation of $f(x) = x^2$ represented by $h(x)$ then write the rule for $h(x)$.

• translated 4 units left + 5 units up.

• reflection over x-axis

$h(x) = -(x+4)^2 + 5$



2. The function g(x) is a transformation of $f(x) = 2(x - 2)^2$ that is translated 3 units to the right, 4 units up, and is reflected over the x-axis. Write a rule for g(x) in vertex form.

transformations

3 right → h+3

4 up → k+4

ref. x-axis → a·-1

$f(x) = 2(x-2)^2 + 0$

$a = 2$ $\quad h = 2$ $\quad k = 0$

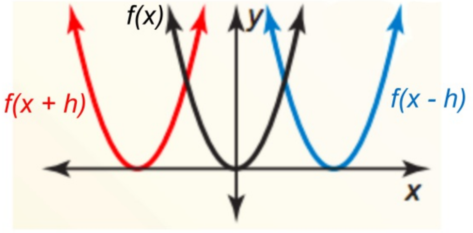
$x -1$ $\quad +3$ $\quad +4$

$a = -2$ $\quad h = 5$ $\quad k = 4$

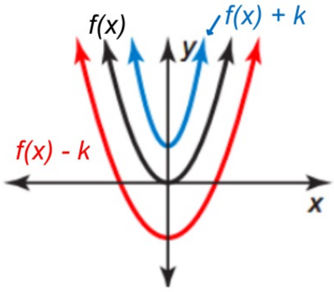
$$\boxed{g(x) = -2(x-5)^2 + 4}$$

Function Notation

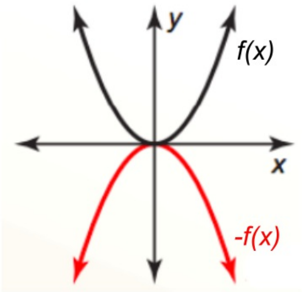
Horizontal Translation: *f(x - h)* slides right
f(x + h) slides left



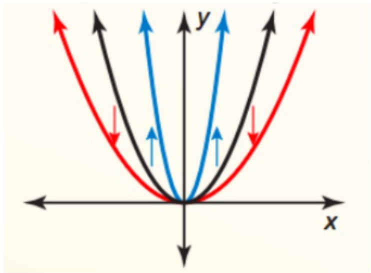
Vertical Translation: *f(x) + k* slides up
f(x) - k slides down



Reflection over the x-axis: *- f(x)*



Vertical Stretch or Compression: *a • f(x)*
a > 0 vertical stretch (narrow)
0 < a < 0 vertical compression (wide)



Examples:

1. The following rule describes the transformation of $f(x) = x^2$ to g(x). Describe the transformation, write the rule for g(x) and graph.

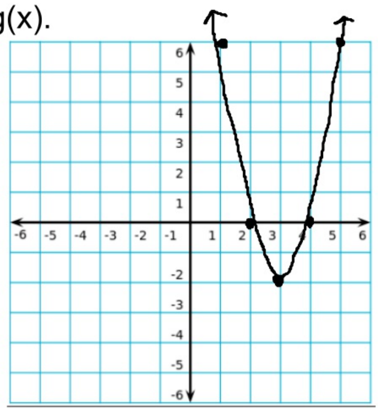
g(x) = 2 • f(x - 3) - 2

vert stretch factor of 2

translated 3 units right

translated 2 units down

$\boxed{g(x) = 2(x-3)^2 - 2}$



2. The following rule describes the transformation of $f(x) = 2x^2 - 4x + 5$ to g(x) and h(x). Describe the transformation, then write the rule.

write in vertex form

$f(x) = 2(x^2 - 2x + \underline{1}) + 5 - \underline{1(2)}$

$f(x) = 2(x-1)^2 + 3$

g(x) = -f(x) + 6

ref. over x-axis
a·-1

trans. 6 up
K+6

a=2 h=1 K=3
x-1 same +6
a=-2 h=1 K=9

$\boxed{g(x) = -2(x-1)^2 + 9}$

h(x) = -0.5f(x + 6)

· vert. comp scale fact .5
· ref. over x-axis
a(-.5)

trans 6 left
h-6

a=2 h=1 K=3
x -.5 -6 same
a=-1 h=-5 K=3

$\boxed{h(x) = -(x+5)^2 + 3}$